5 INTRODUCTION TO EUCLID'S GEOMETRY

EXERCISE 5.1

Q.1. *Which of the following statements are true and which are false? Give reasons for your answers.*

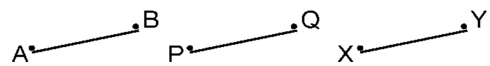
(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In the Fig., if AB = PQ and PQ = XY, then AB = XY.



Ans. (i) False. Infinitely many lines can pass through a point in different directions.

(ii) False. Through two distinct points only one line can pass.

(iii) True. A terminated line or line segment can be produced indefinitely on both sides to give a line.

(iv) True. Two circles of equal area (i.e., equal circles) will have the same radius from the relation area = πr^2.

(v) True. From the axiom that if two things are, separately, equal to a third thing, then, they are equal to each other.

Q.2. *Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?*

(i) parallel lines (ii) perpendicular lines (iii) line segment

(iv) radius of a circle (v) square

Ans. (i) Parallel lines : Two straight lines which have no point in common are said to be parallel to each other.

'Point' and 'straight line' will have to be defined first. 'Point' and 'straight line' as defined in Euclid's text are :

A point is that which has no part.

A line is breadthless length and a straight line is a line which lies evenly with the points on itself.

(ii) Perpendicular lines : If one among two parallel lines is turned by 90°, the two lines become perpendicular to each other. Parallel lines has been defined before, 'rotation through 90° needs further defining. Rotation may be assumed as an intuition therefore, can not be used.

(iii) Line segment : A line with two end points is a line segment.

'Line' and 'point' have been defined before.

(iv) Radius of a circle : The line segment with one end point at the centre and the other at any point on the circle.

'Centre' may be defined (assuming inside) as a point inside the circle which is at the same distance from all points on the circle.

(v) Square : A quadrilateral with all sides equal and all angles right angles is a square.

A quadrilateral is a figure with four sides.

'Figure', 'side' and 'angle' may be assumed known.

Q.3. *Consider two 'postulates' given below :*

(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

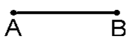
(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Ans. In postulate (i) 'in between A and B' remains an undefined term which appeals to our geometric intuition.

The postulates are consistent. They do not contradict each other. Both of these postulates do not follow from Euclid's postulates However, they follow from the axiom given below.

Given two distinct points, thre is a unique line that passes through them.

(i) Let 

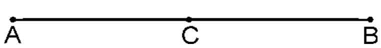
AB be a straight line.

There are an infinite number of points composing this line. Choose any except the two end-points A and B. This point lies between A and B.

(ii) If there are only two points, they can always be connected by a straight line (From Euclid's postulate). Therefore, there have to be at least three points for one of them not to fall on the straight line between the other two.

Q.4. *If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$AB. Explain by drawing the figure.*

Ans.



AC = CB

Also AC + AC = BC + AC. (Equals are added to equals)

BC + AC coincides with AB

$\Rightarrow$ 2AC = AB

$\Rightarrow$ AC = $\frac{1}{2}$AB.

Q.5. *In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.*

Ans. Let there be two such mid points C and D. Then from above theorem

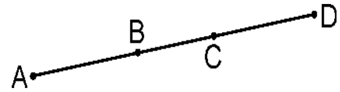
AC = $\frac{1}{2}$AB

and AD = $\frac{1}{2}$AB

$\therefore$ AC = AD

But this is possible only if D coincides with C. Therefore, C is the unique mid-point. **Proved.**

Q.6. *In Fig., if AC = BD, then prove that AB = CD.*



Ans. Given : AC = BD

To prove AB = CD.

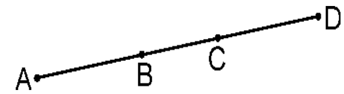
AC = AB + BC

BD = BC + CD

As AC = BD (given)

AB + BC = BC + CD

∴ AB = CD. **Proved.**



Q.7. *Why is Axiom* 5, *in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)*

Ans. **Axiom 5 :** 'Whole is always greater than its part.'

This is a 'universal truth' because part is included in the whole and therefore can never be greater than the whole in magnitude.

5 INTRODUCTION TO EUCLID'S GEOMETRY

EXERCISE 5.2

Q.1. *How would you rewrite Euclid's fifth postulate so that it would be easier to understand?*

Ans. When two lines are cut by a third line, such that the sum of interior angles is less than 180° on one side then the first two lines intersect on the same side.

Q.2. *Does Euclid's fifth postulate imply the existence of parallel lines? Explain.*

Ans. It may be argued that Euclid's fifth postulate recognises the existence of parallel lines. If the sum of interior angles is 180° on both sides of the transversal then the lines will not intersect on any side (condition for intersection being that the sum of interior angles on that side should be less than 180°). So if two lines never intersect, then they are parallel.